Technical Report 17

PRELIMINARY RESULTS ON FAST LUMINOUS FRONTS IN ELECTROMAGNETIC SHOCK TUBES

By: R. A. Nelson

Prepared for:

DIRECTOR
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STANFORD RESEARCH INSTITUTE

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SRI Project No. 3857

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ABSTRACT

Preliminary results of a theoretical study of precursor effects in electromagnetic shock tubes are presented. In particular, an examination is made of the theories of fast luminous fronts which are observed to precede shock waves in such shock tubes. Errors in a theory by Paxton and Fowler are indicated.

CONTENTS

I INTRODUCTION

This technical report contains some preliminary results of a theoretical study of precursor effects in electromagnetic shock tubes. The study was initiated to supplement the experiment work on electromagnetic shock tubes that is being conducted in the Electromagnetic Sciences Laboratory here at SRI.

Precursor ionization ahead of the first shock in electromagnetic shock tubes has been widely observed. A much less widely observed phenomenon is that of a fast luminous front preceding the first shock. The exact relationship between these two phenomena has yet to be established. This report deals primarily with results obtained on the fast luminous fronts, but the theory suggested for the luminous fronts also accounts for the precursor ionization that results from the driving discharge.

Fast luminous fronts have been observed to precede the first shock wave in electromagnetic shock tubes. Josephson and Hales[1] noted a faintly luminous front in deuterium that traveled at speeds between 30 and 120 cm/μsec at ambient pressures between 0.1 and 2.5 mm Hg. A conical shock tube was driven by a 3.2 μf capacitor charged to 24 kv. They speculated on the possibility that the front was due to deuterons that were accelerated to kilovolt energies by instabilities occurring in discharge. Medford, Powell, and Fletcher[2] also observed a luminous front moving ahead of the first shock in deuterium. The speed of the first front was 5 cm/μsec at an ambient pressure of 4 mm Hg. Energy for their shock tube was supplied by a 10-μf capacitor charged to 10 kv. They concluded that this front had the properties of a weak *R*-type ionization front.[3] In the terminology of fluid dynamics, such a front is classified as a weak detonation front.[4] Fowler and Hood[5] observed fast luminous waves in hydrogen and argon. They reported velocities between 60 and 400 cm/μsec at pressures between 0.1 and 1 mm Hg using voltages from

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[1] V. Josephson and R. W. Hales, Space Technology Laboratories Report STL/TR-60-0000-19313, 1960 (unpublished).

[2] R. D. Medford, A. L. T. Powell and W. H. W. Fletcher, Nature **196**, 32 (1962).

[3] F. D. Kahn, Bull. Astro. Inst. Neth. **12**, 187 (1954).

[4] R. Courant and K. O. Friedrichs, Supersonic Flow and Shock Waves (Interscience Publishers, New York, 1948).

[5] R. G. Fowler and J. D. Hood, Jr., Phys. Rev. **128**, 991 (1962).

3 to 9 kv. A companion paper by Paxton and Fowler[6] presents a theory for breakdown wave propagation and relates such waves with the observed fast luminous fronts.

A criticism of the treatment by Paxton and Fowler will be made in this note, and it will be hypothesized that breakdown waves can be treated as ionization fronts to first order, with electrical current constituting a second-order effect.

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[6] G. W. Paxton and R. G. Fowler, Phys. Rev. **128**, 993 (1962).

II CRITICISM OF PAXTON-FOWLER THEORY

Paxton and Fowler[6] consider the propagation of luminosity fronts associated with the electrical breakdown of a gas. A breakdown wave front was treated as an electron shock wave. They presumed that near the electrode where the potential gradient in the gas is greatest, ionization of a small quantity of gas occurs and that the electrons produced are given kinetic energy by the electric field. The resulting localized high-temperature electron gas is considered to expand rapidly, thus producing an electron shock wave which propagates into the ambient gas, partially ionizing the overrun neutral gas molecules. An inconsistency in their paper results from the statement that "the energy necessary for driving the shock wave is given directly to the electrons in the shock zone by the external field;" actually their treatment indicates—as they later state explicitly—that there is only a secondary dependence of the propagation speed on the direct effect of the electric field. The primary driving mechanism in their treatment is the partial pressure of high-temperature electrons behind the front.

They use a one-dimensional, steady-state, three-fluid, hydrodynamical model assuming that the electron pressure is much greater than the partial pressures of the other species, that there is no electrical current, and that there is negligible heat flow. The principal criticism of their paper lies in their treatment of the zero electrical current assumption. As a first-order condition their assumption seems quite reasonable, but they express this condition as follows:

$$nv - N_i V_i = 0 \tag{1}$$

where

- n = Number density of electrons
- v = Flow velocity of electrons in the frame in which the front is at rest
- N_i = Number density of ions
- V_i = Flow velocity of ions in the frame in which the front is at rest.

Actually this equation is the steady-state expression for charge conservation across the front (*i.e.*, ahead of the front there are no electrons or positive ions and at the front the same number of free electrons and singly ionized positive ions are generated; thus $nv = N_i V_i$ both ahead of and behind the front). The condition for zero electrical current should be written in terms of the corresponding velocities in the laboratory frame. Thus

$$n(v - V_0) - N_i(V_i - V_0) = 0 \tag{2}$$

where V_0 is the speed of the front. Except for the trivial case in which $V_0 = 0$, these two equations imply that $N_i = n$ and $V_i = v$. Hence the ions and electrons would travel together, and a three-fluid model is not necessary for a zero-current model.

As a result of the misinterpretation of Eq. (1) the solution given by Paxton and Fowler does have current flowing in the laboratory frame. And since the electron flow is in the same direction as the front velocity while the ions are virtually stationary, this gives the unlikely result that current flows against the electric field for the case of a breakdown wave originating at a positive electrode. On the other hand, a model of this kind may have merit for the case of a precursor wave traveling into an essentially field-free region.

III FIRST-ORDER IONIZATION-FRONT THEORY

An alternative method that appears promising for the treatment of both breakdown waves and precursor waves is to treat these phenomena as ionization fronts, as suggested by Medford, Powell and Fletcher.[2] To first order, the luminous front preceding a shock wave in an electromagnetic shock tube will be considered to be the same as a breakdown wave in a gaseous electrical discharge. It is assumed that the breakdown initiates at the electrode with the greatest potential gradient and that a localized region of hot ionized gas is formed. Ionizing radiation from this hot gas is assumed to be the primary driving mechanism for an ionization front that moves out from the hot gas. It is further assumed that to first order there is no electrical current; hence, a single-fluid model will be used.

Consider a one-dimensional picture. Monochromatic ionizing radiation is coming from the negative x-direction and is being absorbed at the ionization front. An ionization front consists of a comparatively thin photoabsorbing region situated between transparent ionized gas behind the front and opaque un-ionized gas ahead of the front. The front moves in the positive x-direction into the un-ionized gas at a velocity, V_0, that is determined by the intensity of the ionizing radiation.

The ratio of specific heats will be taken to be $5/3$ (*i.e.*, monatomic molecules are assumed), so the speed of sound, c_0, in the ambient gas is $[(5\,p_0)/(3\,\rho_0)]^{1/2}$ where p_0 is the ambient pressure and ρ_0 is the ambient density. The one-dimensional fluid-dynamic equations for transfer of mass, momentum, and energy in the rest frame of the front are

$$\rho_1 v_1 = \rho_0 V_0 \tag{3}$$

$$p_1 + \rho_1 v_1^2 = p_0 + \rho_0 V_0^2 \tag{4}$$

$$\frac{5}{2}\frac{p_1}{\rho_1} + \frac{1}{2}v_1^2 = \frac{5}{2}\frac{p_0}{\rho_0} + \frac{1}{2}V_0^2 + \frac{1}{2}Q^2 \tag{5}$$

where $(1/2)Q^2$ is the excess kinetic energy per unit mass available after ionization of an atom, and is defined by

$$\frac{1}{2} mQ^2 = h\nu - E_i \tag{6}$$

with m = mass of an atom, h = Planck's constant, ν = frequency of the ionizing radiation, and E_i = ionization energy.

Define

$$\epsilon = \frac{\rho_1}{\rho_0} \quad . \tag{7}$$

Then by Eqs. (3), (4), and (5) the equation for ϵ is

$$\left[\frac{5p_0}{\rho_0} + V_0^2 + Q^2\right]\epsilon^2 - 5\left[\frac{p_0}{\rho_0} + V_0^2\right]\epsilon + 4V_0^2 = 0 \tag{8}$$

or, in terms of the speed of sound in the ambient gas,

$$(3c_0^2 + V_0^2 + Q^2)\epsilon^2 - (3c_0^2 + 5V_0^2)\epsilon + 4V_0^2 = 0 \quad . \tag{9}$$

Kahn[3] has enumerated the possible solutions of Eq. (9) and related them to different kinds of fronts. The kind of front that is applicable here is a weak R-type ionization front. It is characterized by a speed of propagation that is supersonic both with respect to the ambient gas ahead of the front and the ionized gas behind the front. A necessary condition thus obtained from Eq. (9) for the existence of such a front is

$$V_0 > \frac{1}{3} \left[2Q + (4Q^2 + 9c_0^2)^{1/2}\right] \quad . \tag{10}$$

In practice this condition is easily met since ionization of molecules typically requires radiation at wavelengths of many hundreds of Angstroms while for Q to be of comparable magnitude to V_0 at 10^8 cm/sec, radiation near one Angstrom would be required. Any source with most of its ionizing radiation at wavelengths greater than 1 Å would certainly fulfill this condition. Actually one would expect that $V_0 \gg Q$, c_0 so that $\epsilon \doteq 1$ and $v_1 \doteq V_0$. Thus in the laboratory frame the magnitude of flow speed of the ionized fluid would be much smaller than the rate of advance of the ionization front, so that to a good approximation the flow speed of the ionized fluid is zero.

A model of such an ionization front, for which the shape of the ionization density contour is easily calculable, is as follows: Consider a one-dimensional model in which a source of ionizing radiation with a flux J_0 photons/cm^2 sec is located at $x = -\infty$. Assume that only the neutral molecules are effective in the absorption of radiation, that the absorption of each photon gives rise to a single ion-electron pair and that the argument of all dependent variables is given by $\zeta = x - V_0 t$ for t finite. The equation for the absorption of the radiation flux is given by

$$\frac{\partial J(\zeta)}{\partial \zeta} = -\alpha J(\zeta) n_n(\zeta) \qquad (11)$$

where α is the absorption coefficient and $n_n(\zeta)$ is the number density of neutral molecules. Using the boundary condition $J(-\infty) = J_0$ gives

$$J(\zeta) = J_0 \exp\left\{-\alpha \int_{-\infty}^{\zeta} n_n(z)dz\right\} . \qquad (12)$$

Let the ambient density of neutral molecules be $n_0 = n_n(\infty)$ and define $\zeta = 0$ by the condition $n_n(0) = (n_0/2)$. Then

$$J(\zeta) + V_0 n_n(\zeta) = J_0 \qquad (13)$$

and the speed of the front is

$$V_0 = \frac{J_0}{n_0} . \qquad (14)$$

From Eqs. (12), (13), and (14), the integral equation for the shape of the front is

$$n_0 = n_n(\zeta) + n_0 \exp\left\{-\alpha \int_{-\infty}^{\zeta} n_n(z)dz\right\} \qquad (15)$$

with $\alpha > 0$, one obtains

$$n_n(\zeta) = \frac{n_0}{1 + \exp(-\alpha n_0 \zeta)} \tag{16}$$

for the neutral molecules. Thus the number density of the ions and electrons is given by

$$n_i(\zeta) = n_e(\zeta) = \frac{n_0}{1 + \exp(\alpha n_0 \zeta)} .$$

IV CONCLUDING REMARKS

It appears that the principal difference between a breakdown wave moving between a pair of electrodes and a fast luminous front moving away from the driving discharge in an electromagnetic shock tube may lie in the presence or absence of current flow, respectively. As a breakdown wave moves out from the initiating electrode, the electrode is in effect being extended into the gas. Thus there is a redistribution of surface charge over the advancing surface of the ionization front. This current flow is relatively small and can be considered as a second-order effect.

It has been assumed in the foregoing argument that radiation from the hot gas is the driving mechanism. Soft X-ray emission due to bombardment of the initiating electrode can also provide a contribution.

Medford, Powell, and Fletcher[2] calculate that an ionizing photon flux of 3.5×10^{23} photons/cm^2 sec is required to account for a speed of 5×10^5 cm/sec at a pressure of 4 mm Hg. It is interesting to note that this same flux would give a speed of about 1.1×10^8 cm/sec at 0.1 mm Hg, which gives order-of-magnitude agreement with the observations of Josephson and Hales,[1] as well as Fowler and Hood.[5]

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